Taubin's algebraic fit
(3D quadric surface)

$C^T = [c_0, c_1, \ldots, c_9]$

$V^T = [1, x, y, z, xy, xz, yz, x^2, y^2, z^2]$ = „funckia" of $\vec{x}$

$f(\vec{x}) = C^T V = V^T C$

n - points @ V_i coords

$$\arg\min_{C} \frac{\sum_{i=1}^{n} (C^T V_i)^2}{\sum_{i=1}^{n} \|\nabla (C^T V_i)\|^2}$$

$$\sum_{i=1}^{n} (C^T M_i C) = C^T \sum_{i=1}^{n} M_i C = C^T M C$$

10×10

$$V_i V_i^T = M_i = \begin{pmatrix} 1 \cdot V_i \\ x\, V_i \\ y\, V_i \\ \vdots \\ z^2 V_i \end{pmatrix} \Rightarrow M = \begin{pmatrix} n & \sum x_i & \cdots & \sum z_i^2 \\ \sum x_i & \sum x_i^2 & & \sum z_i^2 x_i \\ \sum y_i & \sum x_i y_i & \cdots & \sum z_i^2 y_i \\ \vdots & \vdots & & \vdots \\ \sum z_i^2 & \sum x_i z_i^2 & & \sum z_i^4 \end{pmatrix}$$

$M = \sum_i (V_i V_i^T)$

$$\nabla (C^T V_i) = \begin{bmatrix} c_1 + y_i c_4 + z_i c_5 + 2x_i c_7, \\ c_2 + x_i c_4 + z_i c_6 + 2y_i c_8, \\ c_3 + x_i c_5 + y_i c_6 + 2z_i c_9 \end{bmatrix} \begin{matrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{matrix} = \begin{bmatrix} P_{i1}, \\ P_{i2}, \\ P_{i3} \end{bmatrix}$$

$\|\nabla (C^T V_i)\|^2 = P_{i1}^2 + P_{i2}^2 + P_{i3}^2$

$V_1 = [0, 1, 0, 0, y, z, 0, 2x, 0, 0]^T$

$V_2 = [0, 0, 1, 0, x, 0, z, 0, 2y, 0]^T$

$V_3 = [0, 0, 0, 1, 0, x, y, 0, 0, 2z]^T$

$\sum_i \|\ldots\|^2 = \sum_i P_{i1}^2 + \sum_i P_{i2}^2 + \sum_i P_{i3}^2$

$P_{i1} = C^T V_{i1} \qquad P_{i1}^2 = C^T V_{i1} V_{i1}^T C$

$$\sum_i \|\ldots\|^2 = C^T N C, \quad N = \sum_{i=1}^{n} (V_{i1} V_{i1}^T + V_{i2} V_{i2}^T + V_{i3} V_{i3}^T)$$