

MASARYK UNIVERSITY  
FACULTY OF INFORMATICS



# The Proof of $P = NP$

BACHELOR'S THESIS

**Jane Doe**

Brno, Fall 1999



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*This is where a copy of the official signed thesis assignment and a copy of the Statement of an Author is located in the printed version of the document.*



## **Declaration**

Hereby I declare that this paper is my original authorial work, which I have worked out on my own. All sources, references, and literature used or excerpted during elaboration of this work are properly cited and listed in complete reference to the due source.

Jane Doe

**Advisor:** John Smith



## **Acknowledgement**

This is the acknowledgement for my thesis, which can span multiple paragraphs.

## **Abstract**

This is the abstract of my thesis, which can span multiple paragraphs.

## **Keywords**

keyword1, keyword2, ...



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# 1 Introduction

Theses are rumoured to be the capstones of education, so I decided to write one of my own. If all goes well, I will soon have a diploma under my belt. Wish me luck!

Říká se, že závěrečné práce jsou vyvrcholením studia a tak jsem se rozhodl jednu také napsat. Pokud vše půjde podle plánu, odnesu si na konci semestru diplom. Držte mi palce!

Hovorí sa, že záverečné práce sú vyvrcholením štúdia a tak som sa rozhodol jednu tiež napísať. Ak všetko pôjde podľa plánu, odnesiem si na konci semestra diplom. Držte mi palce!

Man munkelt, dass die Dissertation die Krönung der Ausbildung ist. Deshalb habe ich mich beschlossen meine eigene zu schreiben. Wenn alles gut geht, bekomme ich bald ein Diplom. Wünsch mir Glück!

Говорят, что тезис – это кульминация обучения. Поэтому я и решил написать собственный тезис. Если всё сработает по плану, я скоро получу диплом. Желайте мне удачи!



## 2 These are

### 2.1 the available

#### 2.1.1 sectioning

commands.

#### Paragraphs and

**subparagraphs are available as well.** Inside the text, you can also use unnumbered lists,

- such as
- this one
  - and they can be nested as well.
  - » You can even turn the bullets into something fancier,  
§ if you so desire.

Numbered lists are

1. very
  - (a) similar

and so are description lists:

**Description list** A list of terms with a description of each term

The spacing of these lists is geared towards paragraphs of text. For lists of words and phrases, the paralist package offers commands

- that
  - are
  - \* better
  - suited
- 1. to
  - (a) this
    - i. kind of

A. content.

The amsthm package provides the commands necessary for the type-setting of mathematical definitions, theorems, lemmas and proofs.

**Theorem 2.1.1.** *This is a theorem that offers a profound insight into the mathematical sectioning commands.*

**Theorem 2.1.2** (Another theorem). *This is another theorem. Unlike the first one, this theorem has been endowed with a name.*

**Lemma 2.1.3.** *Let us suppose that  $x^2 + y^2 = z^2$ . Then*

$$\left\langle u \left| \sum_{i=1}^n F(e_i, v) e_i \right. \right\rangle = F\left(\sum_{i=1}^n \langle e_i | u \rangle e_i, v\right). \quad (2.1)$$

*Proof.*  $\nabla^2 f(x, y) = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$ . □

**Corollary 2.1.4.** *This is a corollary.*

*Remark.* This is a remark.

### 3 Floats and references

The logo of the Masaryk University is shown in Figure 3.1 and Figure 3.2 at pages 5 and 6. The weather forecast is shown in Table 3.1 at page 6. The following chapter is Chapter 4 and starts at page 7. Items 3, 3b, and 3(c)iv are starred in the following list:

1. some text
2. some other text
3. ★
  - (a) some text
  - (b) ★
  - (c) some other text
    - i. some text
    - ii. some other text
    - iii. yet another piece of text
    - iv. ★
  - (d) yet another piece of text
4. yet another piece of text

If your reference points to a place that has not yet been typeset, the `\ref` command will expand to `??` during the first run of `pdflatex fi-pdflatex.tex` and a second run is going to be needed for the references to resolve. With online services – such as Overleaf – this is performed automatically.



Figure 3.1: The logo of the Masaryk University at 40 mm



Figure 3.2: The logo of the Masaryk University at  $\frac{2}{3}$  and  $\frac{1}{3}$  of text width

Day	Min Temp	Max Temp	Summary
Monday	13°C	21°C	A clear day with low wind and no adverse current advisories.
Tuesday	11°C	17°C	A trough of low pressure will come from the north-west.
Wednesday	10°C	21°C	Rain will spread to all parts during the morning.

Table 3.1: A weather forecast

## 4 Mathematical equations

$\text{\TeX}$  comes pre-packed with the ability to typeset inline equations, such as  $e^{ix} = \cos x + i \sin x$ , and display equations, such as

$$\mathbf{A}^{-1} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{\det(\mathbf{A})} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}.$$

$\text{\LaTeX}$  defines the automatically numbered equation environment:

$$\gamma Px = PAx = PAP^{-1}Px. \quad (4.1)$$

The package `amsmath` provides several additional environments that can be used to typeset complex equations:

1. An equation can be spread over multiple lines using the `multline` environment:

$$\begin{aligned} a + b + c + d + e + f + b + c + d + e + f + b + c + d + e + f \\ + f + g + h + i + j + k + l + m + n + o + p + q \end{aligned} \quad (4.2)$$

2. Several aligned equations can be typeset using the `align` environment:

$$a + b = c + d \quad (4.3)$$

$$u = v + w + x \quad (4.4)$$

$$i + j + k + l = m \quad (4.5)$$

3. The `alignat` environment is similar to `align`, but it doesn't insert horizontal spaces between the individual columns:

$$a + b + c + d \quad = 0 \quad (4.6)$$

$$e + f + g = 5 \quad (4.7)$$

4. Much like chapter, sections, tables, figures, or list items, equations – such as (4.8) and (My equation) – can also be labeled and referenced:

$$b_{11}x_1 + b_{12}x_2 + b_{13}x_3 \quad = y_1, \quad (4.8)$$

$$b_{21}x_1 + b_{22}x_2 \quad + b_{24}x_4 = y_2. \quad (\text{My equation})$$

#### 4. MATHEMATICAL EQUATIONS

---

5. The `gather` environment makes it possible to typeset several equations without any alignment:

$$\psi = \psi\psi, \tag{4.9}$$

$$\eta = \eta\eta\eta\eta\eta\eta, \tag{4.10}$$

$$\theta = \theta. \tag{4.11}$$

6. Several cases can be typeset using the `cases` environment:

$$|y| = \begin{cases} y & \text{if } z \geq 0, \\ -y & \text{otherwise.} \end{cases} \tag{4.12}$$

For the complete list of environments and commands, consult the `amsmath` package manual<sup>1</sup>.

---

1. See <http://mirrors.ctan.org/macros/latex/required/amslatex/math/amslatex.pdf>. The `\url` command is provided by the package `url`.

## 5 We have several FONTS *at disposal*

The serifed roman font is used for the main body of the text. *Italics are typically used to denote emphasis or quotations.* The teletype font is typically used for source code listings. The **bold**, SMALL-CAPS and sans-serif variants of the base roman font can be used to denote specific types of information.

We can also change the font size, although it is usually not necessary.

A wide variety of mathematical fonts is also available, such as:

ABC, *ABC*, **ABC**, ABC, ABC, ABC

By loading the amsfonts packages, several additional fonts will become available:

ℳℳℳ, ABC

Many other mathematical fonts are available<sup>1</sup>.

---

1. See <http://tex.stackexchange.com/a/58124/70941>.



## 6 Using lightweight markup

If you decide that L<sup>A</sup>T<sub>E</sub>X is too wordy for some parts of your document, there are packages<sup>1</sup> that allow you to use more lightweight markup next to it.

This is a bullet list. Unlike numbered lists, bulleted lists contain an *unordered* set of bullet points. When a bullet point contains multiple paragraphs, the list is typeset as follows:

- The first item of a bullet list  
that spans several paragraphs,
- the second item of a bullet list,
- the third item of a bullet list.

When none of the bullet points contains multiple paragraphs, the list has a more compact form:

- The first item of a bullet list,
- the second item of a bullet list,
- the third item of a bullet list.

Unlike a bulleted list, a numbered list implies chronology or ordering of the bullet points. When a bullet point contains multiple paragraphs, the list is typeset as follows:

1. The first item of an ordered list  
that spans several paragraphs,
2. the second item of an ordered list,
3. the third item of an ordered list.
4. If you are feeling lazy,
5. you can use hash enumerators as well.

When none of the bullet points contains multiple paragraphs, the list has a more compact form:

---

1. Markdown: <<https://www.ctan.org/pkg/markdown>>



Figure 6.1: The logo of the Masaryk University

6. The first item of an ordered list,
7. the second item of an ordered list,
8. the third item of an ordered list.

Definition lists are used to provide definitions of terms. When a definition contains multiple paragraphs, the list is typeset as follows:

**Term 1** Definition 1

*Term 2* Definition 2

Some code, part of Definition 2

Third paragraph of Definition 2.

When none of the bullet points contains multiple paragraphs, the list has a more compact form:

**Term 1** Definition 1

*Term 2* Definition 2

Block quotations are used to include an excerpt from an external document in way that visually clearly separates the excerpt from the rest of the work:

This is the first level of quoting.

This is nested blockquote.

Back to the first level.

Footnotes are used to include additional information to the document that are not necessary for the understanding of the main text. Here is a footnote reference<sup>2</sup> and another<sup>3</sup>.

Code blocks are used to include source code listings into the document:

---

2. Here is the footnote.

3. Here's one with multiple blocks.

Subsequent paragraphs are indented to show that they belong to the previous footnote.

Some code

The whole paragraph can be indented, or just the first line. In this way, multi-paragraph footnotes work like multi-paragraph list items.

## 6. USING LIGHTWEIGHT MARKUP

---

```
#include <stdio.h>
#include <unistd.h>
#include <sys/types.h>
#include <sys/wait.h>
// This is a comment
int main(int argc, char **argv)
{
    while (-c > 1 && !fork());
    sleep(c = atoi(v[c]));
    printf("%d\n", c);
    wait(0);
    return 0;
}
```

There is an alternative syntax for code blocks that allows you to specify additional information, such as the language of the source code. This information can be used for syntax highlighting:

```
#!/bin/sh
fac() {
    if [ "$1" -leq 1 ]; then
        echo 1
    else
        echo $(( "$1" * fac $(( "$1" - 1 ))) )
    fi
}
```

```
# Here's a way to empty an array.
joe = [ 'eggs.', 'some', 'break', 'to', 'Have' ]
print(joe.pop, "□") while joe.size > 0
print "\n"
```

## 7 Inserting the bibliography

After linking a bibliography database files to the document using the `\thesissetup{bib={file1,file2,...}}` command, you can start citing the entries. This is just dummy text [1] lightly sprinkled with citations [2, p. 123]. Several sources can be cited at once: [3, 4, 5]. “Fundamental Algorithms” was written by Knuth in 1973. We can also produce Knuth [1] or (Knuth [1], 1973). The full bibliographic citation is: *KNUTH, Donald E. Fundamental Algorithms. In: Second. Reading, Massachusetts: Addison-Wesley, 1973, vol. 1, chap. 1.2, pp. 10–119. The Art of Computer Programming. This is a full INBOOK entry.* We can easily insert a bibliographic citation into the footnote<sup>1</sup>.

The `\nocite` command will not generate any output, but it will insert its arguments into the bibliography. The `\nocite{*}` command will insert all the records in the bibliography database file into the bibliography. Try uncommenting the command and watch the bibliography section come apart at the seams.

When typesetting the document for the first time, citing a work will expand to [work] and the `\printbibliography` command will produce no output. It is now necessary to generate the bibliography by running `biber fi-pdflatex.bcf` from the command line and then by typesetting the document again twice. During the first run, the bibliography section and the citations will be typeset, and in the second run, the bibliography section will appear in the table of contents.

The `biber` command needs to be executed from within the directory, where the  $\LaTeX$  source file is located. In Windows, the command line can be opened in a directory by holding down the Shift key and by clicking the right mouse button while hovering the cursor over a directory. Select the Open Command Window Here option in the context menu that opens shortly afterwards.

With online services – such as Overleaf – all commands are executed automatically. When you omit the `\printbibliography` command, its location will be decided by the template.

---

1. KNUTH, Donald E. Fundamental Algorithms. In: Second. Reading, Massachusetts: Addison-Wesley, 1973, vol. 1, chap. 1.2, pp. 10–119. The Art of Computer Programming. This is a full INBOOK entry.



## Bibliography

1. KNUTH, Donald E. Fundamental Algorithms. In: Second. Reading, Massachusetts: Addison-Wesley, 1973, vol. 1, chap. 1.2, pp. 10–119. The Art of Computer Programming. This is a full INBOOK entry.
2. LINCOLL, Daniel D. Semigroups of Recurrences. In: LIPCOLL, David J.; LAWRIE, D. H.; SAMEH, A. H. (eds.). *High Speed Computer and Algorithm Organization*. Third. New York: Academic Press, 1977, chap. 3, pp. 179–183. Fast Computers, no. 23. This is a full INCOLLECTION entry.
3. OAHO, Alfred V.; ULLMAN, Jeffrey D.; YANNAKAKIS, Mihalis. On Notions of Information Transfer in VLSI Circuits. In: OZ, Wizard V.; YANNAKAKIS, Mihalis (eds.). *Proc. Fifteenth Annual ACM Symposium on the Theory of Computing*. Boston: Academic Press, 1983, pp. 133–139. All ACM Conferences, no. 17. This is a full INPROCEEDINGS entry.
4. MANMAKER, Larry. *The Definitive Computer Manual*. Silver. Silicon Valley, 1986. This is a full MANUAL entry.
5. MASTERLY, Édouard. *Mastering Thesis Writing*. English Department, 1988. Master's project. Stanford University. This is a full MASTERS-THESIS entry.
6. KNVTH, Jill C. *The Programming of Computer Art* [Vernier Art Center]. Stanford, California, 1988. This is a full BOOKLET entry.



## 8 Inserting the index

After using the `\makeindex` macro and loading the `makeidx` package that provides additional indexing commands, index entries can be created by issuing the `\index` command. It is possible to create ranged index entries, which will encompass a span of text. To insert complex typographic material – such as  $\alpha$  or  $\TeX$  – into the index, you need to specify a text string, which will determine how the entry will be sorted. It is also possible to create hierarchical entries.

After typesetting the document, it is necessary to generate the index by running

```
texindy -I latex -C utf8 -L <locale> fi-pdflatex.idx
```

from the command line, where *<locale>* corresponds to the main locale of your thesis – such as `english`, and then typesetting the document again.

The `texindy` command needs to be executed from within the directory, where the  $\LaTeX$  source file is located. In Windows, the command line can be opened in a directory by holding down the Shift key and by clicking the right mouse button while hovering the cursor over a directory. Select the Open Command Window Here option in the context menu that opens shortly afterwards.

With online services – such as Overleaf – the commands are executed automatically, although the locale may be erroneously detected, or the `makeindex` tool (which is only able to sort entries that contain digits and letters of the English alphabet) may be used instead of `texindy`. In either case, the index will be ill-sorted.



## Index

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## **A An appendix**

Here you can insert the appendices of your thesis.